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Entropy production calculation for turbulent shear flows and their implementation in cfd codes

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Abstract

Entropy production in turbulent shear flows with heat transfer is calculated locally and afterwards integrated over the whole flow domain. This quantity can serve as a parameter to determine the efficiency of turbulent heat transfer processes. Based on the time averaged entropy balance equation, four different mechanisms of entropy production can be identified and cast into mathematical equations. They are: dissipation in the mean and the fluctuating velocity fields and heat flux due to the mean and the fluctuating temperature fields.

It turns out that no additional balance equation has to be solved, provided the turbulent dissipation rate is known in the flow field together with the mean velocity and temperature distribution. Since all four entropy production rates show very steep gradients close to the wall numerical solutions are far more effective with wall functions for the production terms. These wall functions are mandatory when high Reynolds number turbulent models are used, as for example the high Reynolds number k–ε model, like in our case. As an example, flow through a heated pipe with a twisted tape inserted is calculated in detail including the local entropy production rate. For this configuration experimental results show an increase in heat transfer as well as in pressure drop when the spiral slope of the twisted tape is increased. Therefore, no optimum of the spiral slope can be found in the experiments. An analysis based on entropy production, however, reveals that there is a distinct optimum for a certain slope of the twisted tape. Thus, entropy production can be used as an efficiency parameter with respect to minimizing the loss of available work in a process.

Keywords: Entropy production; Turbulent flow; Wall functions

1. Introduction

Numerical prediction of heat transfer rates in turbulent shear flows has attracted considerable attention over the past two decades. Pressure drop and heat transfer predictions often are accurate even in complex geometries. Thus, computational fluid dynamics (CFD) has become state of the art in thermal engineering like in heat-exchanger design. However, all these CFD models only take into account the first law of thermodynamics.

Predicting an efficient use of energy in thermal systems like compact heat exchangers and power plants, however, can only be achieved if also the second law of thermodynamics is accounted for, since the amount of available work (also called exergy) is linked to the amount of entropy produced, see Bejan (1996). Therefore, a thermal apparatus producing less entropy by irreversibilities destructs less available work (producing less anergy). This increases the total efficiency of a thermal system. The amount of entropy produced can be used directly as a parameter to assess the efficiency of the system, see Bejan (1978, 1979, 1980).

The foundation for a second law analysis of processes has been layed by various thermodynamicists like

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Nomenclature

Turbulent Eckert number $Ec_\tau = u_\tau^2/(c_p T_\tau)$ (–)
Turbulent Reynolds number $Re_\tau = u_\tau L/\nu$ (–)
Friction temperature $T_\tau = -q_w/(\varrho c_p u_\tau)$ (K)
Friction velocity $u_\tau = \sqrt{\tau_w/\varrho}$ (m/s)
Dimensionless entropy production rate $S_{\mathrm{PRO}}^{+} = S_{\mathrm{PRO}} \cdot [\nu a (T_w/T_\tau)^2]/[u_\tau^2 \lambda]$ (–)

Onsager (1931) and Prigogine (1978). Until today there is an ongoing discussion about the consequences of the ideas related to the minimization of the rate of entropy production, like in Mahulikar and Herwig (2004).

Second law, and entropy production analysis in particular, have been widely used to assess the sources of irreversibility in components and systems. The majority of these studies, however, are limited to a global analysis. The assessment of local sources of irreversiblities, i.e. local entropy production, is often based on empirical correlations for pressure drop and heat transfer performance. They, however, are known only for special cases, see Zimparov (2000), Sasikumar and Balaji (2002), Gerdov (1996) and Sahin (1998). Other studies concerning local entropy production rates are for laminar flows only, see Abu-Hijleh and Heilen (1999), Abu-Hiljleh et al. (1999), Shuja et al. (1999), Sciubba (1996), Sciubba (1997), Perng and Chu (1995) and Benedetti and Sciubba (1993), or do not include the influence of solid walls in turbulent problems, like Drost and White (1991).

In this paper we present model equations for the calculation of the local entropy production rates in turbulent shear flows by extending the Reynolds-averaging procedure to the entropy balance equation. This equation serves to identify the entropy production sources, without need to solve the equation itself. In the basic equations, including high-Reynolds number k–ε turbulence closure, special attention has been given to the near wall regions where entropy production rates change strongly. Wall functions for all four entropy production terms have been based on asymptotic considerations.

With the set of model equations, local entropy production can be calculated in the post-processing part of a CFD analysis. No further differential- or transport equations need to be solved. Thus, the procedure requires only a small amount of additional CPU time and can easily be implemented in existing CFD codes.

2. Entropy production in turbulent flows

For a systematic derivation of a model for entropy production in turbulent flows with heat transfer we start with the transport equation for entropy (Cartesian coordinates, incompressible fluid, single-phase flow, Fourier heat conduction), see Spurk (1989):

$$\varrho\left(\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z}\right) = \operatorname{div}\left(\frac{\vec{q}}{T}\right) + \boxed{\frac{\Phi}{T}} + \boxed{\frac{\Phi_\Theta}{T^2}} \tag{1}$$

with the dissipation functions Φ and Φ_Θ given in (2)–(5) below. These two terms represent important mechanisms for entropy production, neglecting minor important ones like entropy production due to radiation. Here s is the specific entropy, T the thermodynamic temperature and u, v, w are the Cartesian velocity components.

In (1) the two terms related to entropy production are marked by grey shaded boxes. The first one describes entropy production by viscous dissipation, the second one stands for entropy production by heat transfer due to finite temperature gradients. These terms are always positive and therefore act as source terms in (1). All other terms can be positive or negative, depending on the direction of the flow as well as the heat flux.

For example, a heat transfer apparatus with small cross sections often encounters small temperature differences and therefore small entropy production by heat transfer. However, due to the large pressure drop of this configuration there will be a large entropy production rate by dissipation. Since, however, both effects (heat transfer and pressure drop) have been linked to one single quantity (entropy production), the overall performance can be estimated by the total entropy production rate of the apparatus which should be as small as possible.

If we had not this single quantity, two completely different parameters would serve to find out, for example, if an increase of heat transfer accompanied by an increase of pressure drop is an increase with respect to the overall performance of the apparatus. This, however, would be like comparing apples and pears.

2.1. Time-averaging of the entropy transport equation

Eq. (1) holds for the instantaneous values of specific entropy s, velocities u, v and w and temperature T. According to the RANS (Reynolds averaged Navier–Stokes) approach they are split into time-mean and fluctuating parts, i.e. $s = \bar{s} + s'$, $u = \bar{u} + u'$, ... and inserted into (1). After time averaging, the balance equation for $\bar{s}$ emerges (for more details see Kock, 2003) including the four entropy production terms we are interested in and which will be described next. After time-averaging of Eq. (1) there also appear terms that describe turbulent diffusion of entropy. As long as these terms are unknown the overall amount of entropy production cannot be determined from a simple entropy balance (arguing that entropy is fixed once pressure and temperature are known). For more details of that kind of an alternative approach to determine the overall rate of entropy production see Herwig and Kock (2005).

2.2. Entropy production by dissipation

After time-averaging of the entropy production by dissipation two groups of terms appear, one with mean and one with fluctuating quantities. With $\overline{(\Phi/T)} = S_{\mathrm{PRO},\overline{D}} + S_{\mathrm{PRO},D'}$ they are (μ: dynamic viscosity):

$$S_{\mathrm{PRO},\overline{D}} = \frac{\mu}{\overline{T}} \cdot \left[2\left\{ \left(\frac{\partial \bar{u}}{\partial x}\right)^2 + \left(\frac{\partial \bar{v}}{\partial y}\right)^2 + \left(\frac{\partial \overline{w}}{\partial z}\right)^2 \right\} + \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)^2 + \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \overline{w}}{\partial x}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \overline{w}}{\partial y}\right)^2 \right] \tag{2}$$

$$S_{\mathrm{PRO},D'} = \frac{\mu}{\overline{T}} \cdot \left[2\left\{ \overline{\left(\frac{\partial u'}{\partial x}\right)^2} + \overline{\left(\frac{\partial v'}{\partial y}\right)^2} + \overline{\left(\frac{\partial w'}{\partial z}\right)^2} \right\} + \overline{\left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x}\right)^2} + \overline{\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right)^2} + \overline{\left(\frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y}\right)^2} \right] \tag{3}$$

Here T' in the denominator appears only in higher order terms when $1/T$ is expanded into a series and therefore is neglected in this leading order approach.

The first group represents entropy production by dissipation in the mean flow field, often referred to as *direct dissipation*. The second group of terms then is the so-called *indirect* or *turbulent dissipation*.

2.3. Entropy production by heat transfer

In the heat transfer entropy production term a factor $1/T^2$ appears. Again T' is neglected since it only appears in higher order terms. Entropy production due to heat conduction also has two groups of terms, i.e. $\overline{(\Phi_\Theta/T^2)} = S_{\mathrm{PRO},\overline{C}} + S_{\mathrm{PRO},C'}$ with λ being the thermal conductivity:

$$S_{\mathrm{PRO},\overline{C}} = \frac{\lambda}{\overline{T}^2} \left[\left(\frac{\partial \overline{T}}{\partial x}\right)^2 + \left(\frac{\partial \overline{T}}{\partial y}\right)^2 + \left(\frac{\partial \overline{T}}{\partial z}\right)^2 \right] \tag{4}$$

$$S_{\mathrm{PRO},C'} = \frac{\lambda}{\overline{T}^2} \left[\overline{\left(\frac{\partial T'}{\partial x}\right)^2} + \overline{\left(\frac{\partial T'}{\partial y}\right)^2} + \overline{\left(\frac{\partial T'}{\partial z}\right)^2} \right] \tag{5}$$

The first group of terms is the heat transfer entropy production due to time mean temperature gradients. The second group of terms represents heat transfer entropy production due to fluctuating temperature gradients.

2.4. Interim summary

Altogether four groups of entropy production terms in turbulent flows could be identified in our systematic approach:

1. $S_{\mathrm{PRO},\overline{D}}$: entropy production rate by direct dissipation,
2. $S_{\mathrm{PRO},D'}$: entropy production rate by indirect (turbulent) dissipation,
3. $S_{\mathrm{PRO},\overline{C}}$: entropy production rate by heat conduction due to mean temperature gradients,
4. $S_{\mathrm{PRO},C'}$: entropy production rate by heat conduction due to fluctuating temperature gradients.

Other studies on local entropy production in turbulent flows often are incomplete, for example neglecting $S_{\mathrm{PRO},C'}$, see Drost and White (1991).

As a consequence of the time averaging process new unknowns appear in the equations for the time mean quantities (closure problem). As far as entropy production is concerned they are $S_{\mathrm{PRO},D'}$ and $S_{\mathrm{PRO},C'}$. Note that the entropy balance equation as a whole will not be solved but rather serves to identify all entropy production terms of the problem. Therefore, turbulence modelling is needed only for these terms and will be provided in the following section.

3. Turbulence modelling for entropy production terms

Two of the four entropy production equations are still unclosed ($S_{\mathrm{PRO},D'}$, $S_{\mathrm{PRO},C'}$) and need turbulence modelling. For that purpose information already available in a k–ε turbulence closure of the whole system of equations should be used as far as possible.

3.1. Modelling $S_{PRO,D'}$

The entropy source group $S_{\mathrm{PRO},D'}$ is closely related to the so-called turbulent dissipation rate T_Φ which appears in the k-equation of the k–ε model. It is

$$T_\Phi = \overline{T} \cdot S_{\mathrm{PRO},D'} \tag{6}$$

so that the k–ε model might provide the necessary information to determine $S_{\mathrm{PRO},D'}$.

This model is a two-equation turbulence model based on the equations for the mechanical energy, k, of the velocity fluctuations and a corresponding equation for the dissipation of k, called ε-equation. In order to derive an approximation for the turbulent dissipation rate T_Φ we take a closer look at the transport equation for the mechanical energy of the velocity fluctuations (k-equation, with groups of terms abbreviated as T_{PV1}, T_{PV2}, T_{TD1}, T_{PRO}, T_{VD}, T_Φ see Gersten and Herwig, 1992 for details):

$$\varrho\left(\frac{\partial k}{\partial t}+\bar{u}\frac{\partial k}{\partial x}+\bar{v}\frac{\partial k}{\partial y}+\bar{w}\frac{\partial k}{\partial z}\right) = T_{PV1}-T_{PV2}-T_{\mathrm{TD1}}-T_{\mathrm{PRO}}+T_{\mathrm{VD}}-T_{\Phi} \tag{7}$$

In (7) T_{PV1} and T_{PV2} are pressure/velocity correlations, T_{TD1} is the turbulent diffusion, T_{PRO} the production of turbulent kinetic energy, T_{VD} the viscous diffusion and T_{Φ} the turbulent dissipation. The dissipation rate ε which is used in all standard k–ε models, however, is not equal to the turbulent dissipation rate T_{Φ}/ϱ as one might expect. Instead, some terms of the group T_{VD} in the k-equation are combined with T_{Φ} to a quantity

$$\varepsilon=(T_{\Phi}-T_{\mathrm{VD}}+\mu\Delta k)/\varrho \tag{8}$$

which then explicitly appears in the k-equation after rearranging (7). Here Δ is the Laplace operator.

The benefit is, that now the two terms $(T_{\mathrm{VD}}-T_{\Phi})$ in the k-equation (7) can be replaced by $(\mu\Delta k-\varrho\varepsilon)$, a combination of terms that needs no explicit modelling since it is known in terms of k and ε already. This procedure is standard in all versions of the k–ε model. It should be kept in mind, however, that $\varrho\varepsilon$ is not the exact expression for the turbulent dissipation rate (which is T_{Φ}), and therefore in Gersten and Herwig (1992), for example, is named *pseudo-dissipation*. The difference between $\varrho\varepsilon$ and T_{Φ}, however, is asymptotically small, disappearing for $Re\to\infty$, see Mathieu and Scott (2000). Furthermore, neither k nor ε can be determined exactly from the k- and ε-equations since they contain terms that again need modelling to yield a closed system of equations. Thus, what finally has to be solved are k- and ε-model-equations including a set of (empirical) constants.

From their solution we find the source term $S_{\mathrm{PRO},D'}$ as

$$S_{\mathrm{PRO},D'}=\frac{\varrho\varepsilon}{\overline{T}} \tag{9}$$

3.2. Modelling $S_{PRO,C'}$

If $S_{\mathrm{PRO},C'}$ should be determined like $S_{\mathrm{PRO},D'}$ in Eq. (9), there should be a four equation turbulence model for k, ε, k_{Θ} and ε_{Θ}.

Here, k_{Θ} is the variance of the temperature fluctuations, $k_{\Theta}=\overline{T'^2}/2$, and ε_{Θ} is its dissipation rate. Such models exist, see for example Nagano and Kim (1988), but they are not incorporated in standard CFD-codes. Nevertheless, it is worthwhile to have a closer look at the k_{Θ} equation. It reads, see Gersten and Herwig (1992)

$$\varrho\left(\frac{\partial k_{\Theta}}{\partial t}+\bar{u}\frac{\partial k_{\Theta}}{\partial x}+\bar{v}\frac{\partial k_{\Theta}}{\partial y}+\bar{w}\frac{\partial k_{\Theta}}{\partial z}\right) = T_{\Theta\mathrm{VD}}-T_{\Theta\mathrm{TD}}+T_{\Theta\mathrm{PRO}}-\varrho\varepsilon_{\Theta} \tag{10}$$

Here $T_{\Theta\mathrm{VD}}$ is a group of viscous diffusion terms, $T_{\Theta\mathrm{TD}}$ is turbulent diffusion, $T_{\Theta\mathrm{PRO}}$ production of k_{Θ} and $\varrho\varepsilon_{\Theta}$ its dissipation. The term ε_{Θ} in detail reads

$$\varepsilon_{\Theta}=\alpha\left[\overline{\left(\frac{\partial T'}{\partial x}\right)^2}+\overline{\left(\frac{\partial T'}{\partial y}\right)^2}+\overline{\left(\frac{\partial T'}{\partial z}\right)^2}\right] \tag{11}$$

with the thermal diffusivity α. It is closely related to the entropy production $S_{\mathrm{PRO},C'}$ by

$$\varepsilon_{\Theta}=\frac{\overline{T}^2 S_{\mathrm{PRO},C'}}{\varrho c_p} \tag{12}$$

Since ε_{Θ} is not determined when only a standard two equation k–ε model is used, we alternatively find it from the following considerations.

Except for regions very close to and very far away from a wall, i.e. in the logarithmic region, there often is an equilibrium situation in which production and dissipation of k_{Θ} equal each other in magnitude, i.e. $\varrho\varepsilon_{\Theta}=T_{\Theta\mathrm{PRO}}$, see Eq. (10). Here $T_{\Theta\mathrm{PRO}}$ is

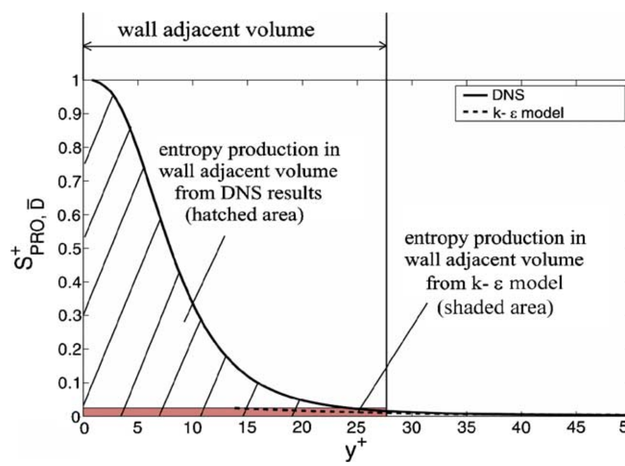

Fig. 1. Entropy production rates by direct dissipation of kinetic energy in the near wall region of a heated channel flow; details in [Kock, 2003], DNS data [Kawamura and Matsuo, 1999].

Table 1
Constants in the wall functions for $S^+_{\mathrm{PRO},\overline{D}}$ and $S^+_{\mathrm{PRO},\overline{C}}$

$a_{\overline{D}}$	$\dfrac{-9PrT_{\tau}y^{+2}_{\ln\overline{D}}}{8T_w-12y^{+}_{\ln\overline{D}}PrT_{\tau}+8\sqrt{T_w^2-3y^{+}_{\ln\overline{D}}PrT_wT_{\tau}}}$
$b_{\overline{D}}$	$\dfrac{8T_w-12y^{+}_{\ln\overline{D}}PrT_{\tau}+8\sqrt{T_w^2-3y^{+}_{\ln\overline{D}}PrT_wT_{\tau}}}{18T_wy^{+2}_{\ln\overline{D}}}$
$A_{\overline{D}}$	$Ec_{\tau}\dfrac{T_w}{T_{\tau}}\exp[b_{\overline{D}}a^2_{\overline{D}}]$
$a_{\overline{C}}$	$\dfrac{-9PrT_{\tau}y^{+2}_{\ln\overline{C}}}{4T_w-12y^{+}_{\ln\overline{C}}PrT_{\tau}+4\sqrt{T_w^2-6y^{+}_{\ln\overline{C}}PrT_wT_{\tau}}}$
$b_{\overline{C}}$	$\dfrac{4T_w-12y^{+}_{\ln\overline{C}}PrT_{\tau}+4\sqrt{T_w^2-6y^{+}_{\ln\overline{C}}PrT_wT_{\tau}}}{9T_wy^{+2}_{\ln\overline{C}}}$
$A_{\overline{C}}$	$Pr\exp[b_{\overline{C}}a^2_{\overline{C}}]$

For T_{τ}, Ec_{τ} see the nomenclature list.

$$T_{\Theta\mathrm{PRO}} = \varrho\left(-\overline{u'T'}\cdot\frac{\partial\overline{T}}{\partial x} - \overline{v'T'}\cdot\frac{\partial\overline{T}}{\partial y} - \overline{w'T'}\cdot\frac{\partial\overline{T}}{\partial z}\right) \qquad (13)$$

When this term can be modelled we can relate it to ε_Θ and thus determine $S_{\mathrm{PRO},C'}$.

Modelling of the terms $\overline{u_i'T'}$ in (13) now is based on the assumption of a constant turbulent Prandtl number $Pr_t = \nu_t/\alpha_t$ and a Boussinesque-like approach $-\overline{u_i'T'} = \alpha_t\partial\overline{T}/\partial x_i$, for the turbulent heat flux. With $\nu_t = C_\mu k^2/\varepsilon$ we thus get

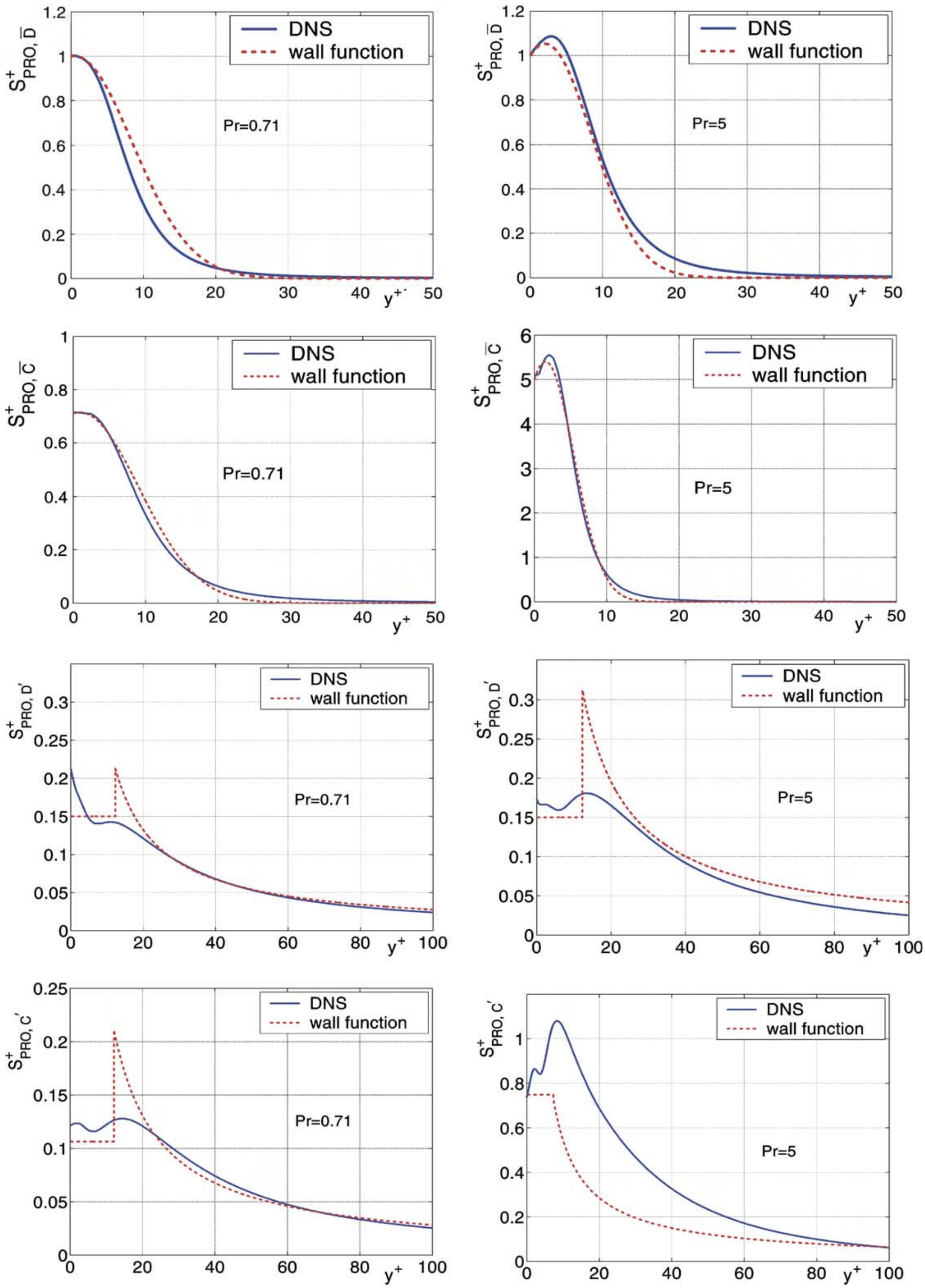

Fig. 2. Wall functions compared to DNS data [Kawamura and Matsuo, 1999]; $Re_\tau = 395$; $Ec_\tau = 0.01$; $T_\tau/T_w = 0.01$.

$$-\overline{u_i'T'} = \frac{\nu_t}{Pr_t}\frac{\partial \overline{T}}{\partial x_i} = \frac{C_\mu}{Pr_t}\frac{k^2}{\varepsilon}\frac{\partial \overline{T}}{\partial x_i} \tag{14}$$

Eq. (14) inserted in (13) with $\varrho\varepsilon_\Theta = T_{\Theta \mathrm{PRO}}$ gives

$$\varepsilon_\Theta = \frac{C_\mu}{Pr_t}\frac{k^2}{\varepsilon}\left[\left(\frac{\partial \overline{T}}{\partial x}\right)^2 + \left(\frac{\partial \overline{T}}{\partial y}\right)^2 + \left(\frac{\partial \overline{T}}{\partial z}\right)^2\right] \tag{15}$$

Our model equation for the source term $S_{\mathrm{PRO},C'}$ therefore is

$$S_{\mathrm{PRO},C'} = \frac{\alpha_t}{\alpha}\frac{\lambda}{\overline{T}^2}\left[\left(\frac{\partial \overline{T}}{\partial x}\right)^2 + \left(\frac{\partial \overline{T}}{\partial y}\right)^2 + \left(\frac{\partial \overline{T}}{\partial z}\right)^2\right] \tag{16}$$

Eq. (16) shows that turbulent entropy production by heat conduction is closely related to the *direct* entropy production by heat conduction, see (4). The only difference in these two equations is a factor α_t/α, which in regions far away from the wall can adopt values of the order 100 or above.

4. Wall functions for entropy production terms

So far, we have shown how all four sources of entropy production in turbulent flows can be calculated in a post-process. The information available in a turbulent heat transfer calculation with k–ε turbulence modelling is sufficient for an a posteriori entropy analysis.

In Kock (2003) the four entropy production terms were determined for a test case (heated turbulent channel flow, $Re = 13.981$, $Pr = 0.71$ and $Pr = 5$) for which DNS data are available in Kawamura and Matsuo (1999). It turns out that for both Prandtl numbers the entropy production rates calculated by the k–ε model equations show good agreement with the DNS calculations provided $y^+ > 50$, i.e. excluding the near wall region. All entropy production rates have peak values near the wall. Especially for the production rates from mean gradients, i.e. $S_{\mathrm{PRO},\overline{D}}$ and $S_{\mathrm{PRO},\overline{C}}$, the model equation results are far off very close to the wall. This is due to the extremely steep gradients of mean velocity and temperature in the immediate vicinity of the wall. In the standard k–ε model they are accounted for by special wall functions.

These wall functions are analytical expressions for the solutions in the wall nearest part of the flow field. They exploit the universal nature of near wall turbulent physics. Thus, there is no need for an extremely fine grid that could resolve the steep gradients that appear close to the wall. Instead, the first finite volume of the numerical grid can be rather large with the analytical solution incorporated.

However, such wall functions have not yet been developed for the entropy production terms, so that errors in these terms are extremely high in the wall nearest volume if it is too big for a resolution of extreme gradients. This is illustrated in Fig. 1 where the wall nearest finite volume extends to $y^+ = 27$. The DNS entropy production rate by direct dissipation (hatched area) by far exceeds that according to the k–ε model equations (shaded area). Obviously, without extra considerations in the wall adjacent volume entropy production rate calculations by the model equations result in unacceptablely large errors. And, one should keep in mind: most of the entropy is generated in the near wall region! This illustrates why wall functions are needed for all four entropy production terms, even for moderate Reynolds numbers. These wall-functions should be analytical over the control volume so that a volume integrated value for the entropy production rate can be determined. Also, they should be consistent with asymptotic representations of the velocity and temperature profiles for $Re \to \infty$ from which entropy production rates for $y^+ \to 0$ and $y^+ \to \infty$ can be found. Here, y^+ is the turbulent wall coordinate yu_τ/ν with $u_\tau = \sqrt{\tau_w/\varrho}$ as skin friction velocity.

Combining asymptotic and DNS considerations, we "construct" wall functions that are asymptotically correct for $y^+ \to 0$ and correspond to DNS results for finite values of y^+. Implicitly we thus assume that the universal nature of wall adjacent functions that is known to exist for $Re \to \infty$ is sufficiently developed already for those Reynolds numbers that can be reached by DNS calculations.

4.1. Wall functions for $S^+_{PRO,\overline{D}}$ and $S^+_{PRO,\overline{C}}$

The general form we assume for these two wall functions with respect to the mean profiles is

$$S^+_{\mathrm{PRO},i} = A_i \exp[-b_i(y^+ - a_i)^2]; \quad i = \overline{D}, \overline{C} \tag{17}$$

In Kock (2003) the constants are determined from asymptotic considerations ($y^+ \to 0$) and DNS data. They are listed in Table 1.

The y^+ values $y^+_{\ln \overline{D}}$ and $y^+_{\ln \overline{C}}$ correspond to the intersection of the asymptotic representations for $y^+ \to 0$ and $y^+ \to \infty$ of the velocity and temperature profiles, respectively. They are $y^+_{\ln \overline{D}} = 11.6$ as well as $y^+_{\ln \overline{C}} = 12.1$ for $Pr = 0.71$ and $y^+_{\ln \overline{C}} = 7.3$ for $Pr = 5$, respectively.

In order to find a mean value of the entropy production in the wall adjacent volume of the finite volume approach, (17) is integrated over this volume. At a distance y^+_{mp} (mp: midpoint, center of the volume) we thus get

$$S^+_{\mathrm{PRO},\overline{D}\mathrm{mp}}(y^+) = \frac{1}{2y^+_{\mathrm{mp}}}\left[\frac{A_{\overline{D}}}{2}\sqrt{\frac{\pi}{b_{\overline{D}}}}\cdot\left[\mathrm{erf}\left(\sqrt{b_{\overline{D}}}2y^+_{\mathrm{mp}} - \sqrt{b_{\overline{D}}}a_{\overline{D}}\right) - \mathrm{erf}\left(-\sqrt{b_{\overline{D}}}a_{\overline{D}}\right)\right]\right] \tag{18}$$

$$S^+_{\mathrm{PRO},\overline{C}\mathrm{mp}}(y^+) = \frac{1}{2y^+_{\mathrm{mp}}}\left[\frac{A_{\overline{C}}}{2}\sqrt{\frac{\pi}{b_{\overline{C}}}}\cdot\left[\mathrm{erf}\left(\sqrt{b_{\overline{C}}}2y^+_{\mathrm{mp}} - \sqrt{b_{\overline{C}}}a_{\overline{C}}\right) - \mathrm{erf}\left(-\sqrt{b_{\overline{C}}}a_{\overline{C}}\right)\right]\right] \tag{19}$$

4.2. Wall functions for $S^+_{PRO,D'}$ and $S^+_{PRO,C'}$

These wall functions are found by patching the asymptotic profiles at their intersection points $y^+_{\ln\overline{D}}$ and $y^+_{\ln\overline{C}}$, respectively.

After an integration over the wall adjacent finite volume the midpoint values for the entropy production are

$$S^+_{\mathrm{PRO},D'\,\mathrm{mp}}(y^+) = \frac{1}{2y^+_{\mathrm{mp}}}\Bigg[0.15Ec_\tau\frac{T_w}{T_\tau}y^+_{\ln\overline{D}} + Ec_\tau\frac{T_w^2}{T_\tau^2}\frac{1}{\kappa} \cdot \Bigg[\log\Bigg\{1+\frac{T_\tau}{T_w}\Big(\log(2y^+_{\mathrm{mp}})+C^+_{\overline{D}}\Big) - \log\Bigg(1+\frac{T_\tau}{T_w}\Big(\log(y^+_{\ln\overline{D}})+C^+_{\overline{D}}\Big)\Bigg)\Bigg\}\Bigg]\Bigg] \tag{20}$$

$$S^+_{\mathrm{PRO},C'\,\mathrm{mp}}(y^+) = \frac{1}{2y^+_{\mathrm{mp}}}\Bigg[0.15Pr y^+_{\ln\overline{C}} + \frac{1}{\frac{T_\tau}{T_w}+\log(y^+_{\ln\overline{C}})+C^+_{\overline{C}}} - \frac{1}{\frac{T_\tau}{T_w}+\log(2y^+_{\mathrm{mp}})+C^+_{\overline{C}}}\Bigg] \tag{21}$$

4.3. Comparison with DNS data

In Fig. 2 all four wall functions are compared to DNS data from Kawamura and Matsuo (1999) for the two Prandtl numbers $Pr = 0.71$ and $Pr = 5$. A detailed discussion of these curves again is found in Kock (2003).

5. An example

An example of a complex convective heat transfer situation is shown in Fig. 3, which shows a pipe of diameter $D = 25.4$ mm and a length of $27.5D$. The turbulent flow of air is heated in the midsection of length $L = 15.5D$ by imposing a constant wall heat flux density $q_w = 8200\ \mathrm{W/m^2}$. In order to increase the heat transfer performance of the pipe, a twisted tape is inserted in the heating section. This leads to a considerable increase of the Nusselt number Nu but also increases the pressure loss coefficient c_f of the whole device. This tape cuts the circular cross section into two half circles which spiral

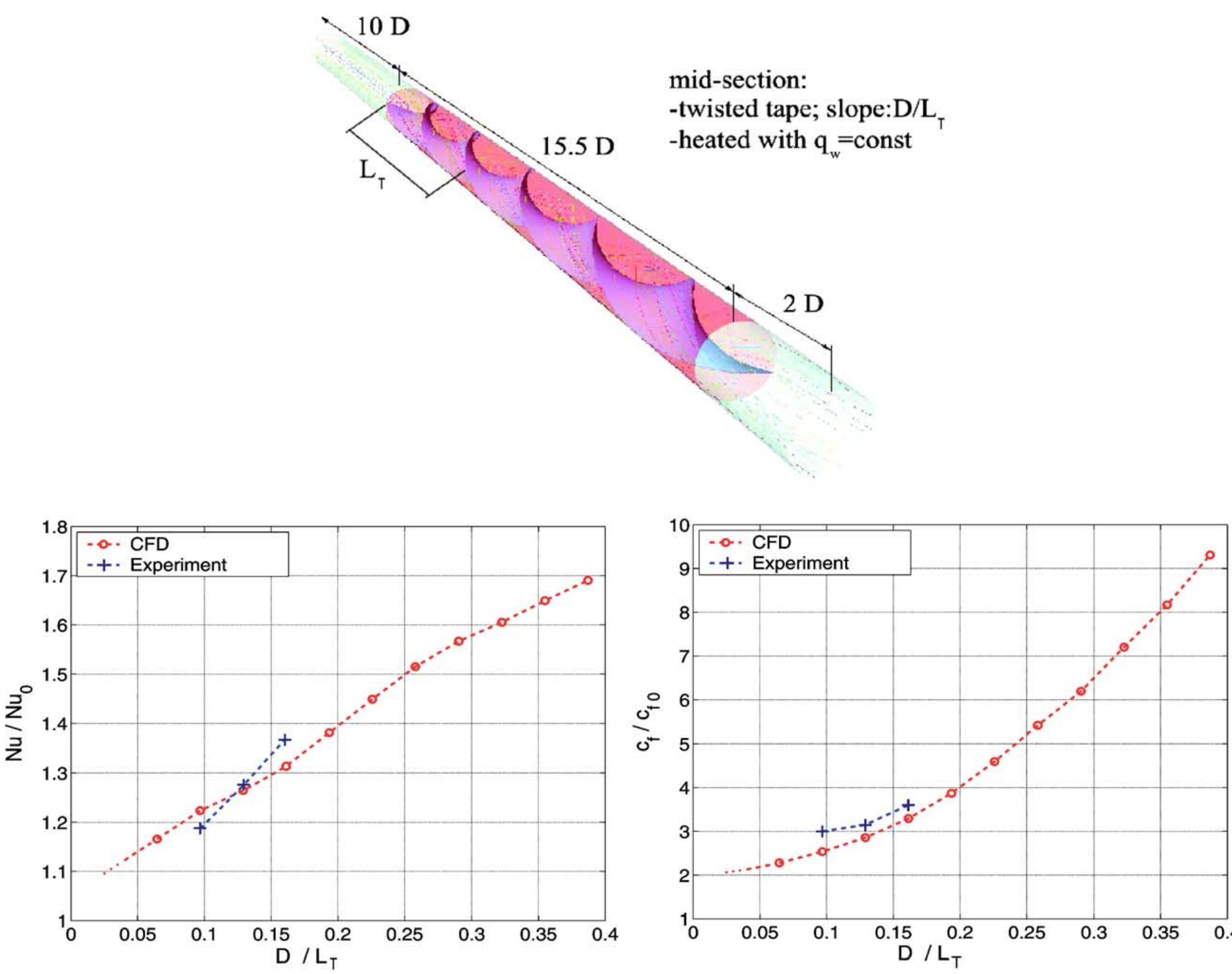

Fig. 3. Heat transfer augmentation by an inserted twisted tape; $Re = u_mD/\nu = 5.1 \times 10^4$; Nu_0, c_{f0}: no tape inserted. CFD [Kock, 2003], Experiment [Zhang et al., 1997].

along the axis of the pipe. The slope of this spiraling is measured in terms of D/L_T where L_T is the length over which the azimuthal angle of the tape changes by 360°.

The wanted increase in heat transfer and the unwanted increase in pressure loss are shown in Fig. 3 for different slopes of the twisted tape, i.e. for different values of D/L_T. Experimental results from Zhang et al. (1997) compare quite well with numerical calculations performed with the finite volume code CFX and a standard high Reynolds number k–ε turbulence model. For our numerical solutions we used the ANSYS CFX 4.2 code. As a result of our grid refinement studies we used 150 finite volumes in streamwise direction for the midsection part of the flow field. In the cross section a distribution of 1776 finite volumes was used with smaller volumes close to the wall. Calculations were stopped when mass and energy source residuals were reduced to 5×10^{-9} and 5×10^{-8}, respectively.

5.1. The evaluation problem

Since both dimensionless parameters, Nu and c_f, increase monotonically for increasing D/L_T, there is no optimum discernable. And, a more fundamental question arises (without answer, so far): should one insert the twisted tape, since for example for $D/L_T = 0.2$ the Nusselt number is increased by 40%, but at the expense of a 400% increase of c_f!

Both effects, the wanted and the unwanted, can simultaneously be evaluated by calculating the overall entropy production of the system. If S_{PRO} increases after a certain change in the system this change is counterproductive, otherwise it is beneficial in the sense that less entropy production means a reduced loss of available work.

From a CFD calculation of the flow and temperature fields all mean gradients of velocity and temperature are known for the problem together with the turbulent (pseudo-)dissipation rate ε. Therefore the entropy production rate can be calculated in a post-process. This has been done for the twisted tape problem using the wall functions deduced in the previous section. Details of these calculations and especially how viscous dissipation and heat conduction contribute to the overall entropy production can be found in Kock (2003). Here we only take these results to demonstrate that they can be used to assess the overall efficiency of inserting twisted tapes.

Fig. 4 now can give a clear answer whether it makes sense to insert the tape or not:

- yes, since the overall entropy production is reduced for a certain range of D/L_T,
- D/L_T should be ≈ 0.18, since then the decrease in entropy production is the strongest. It is almost 8%, i.e. it saves that percentage of exergy (available work).

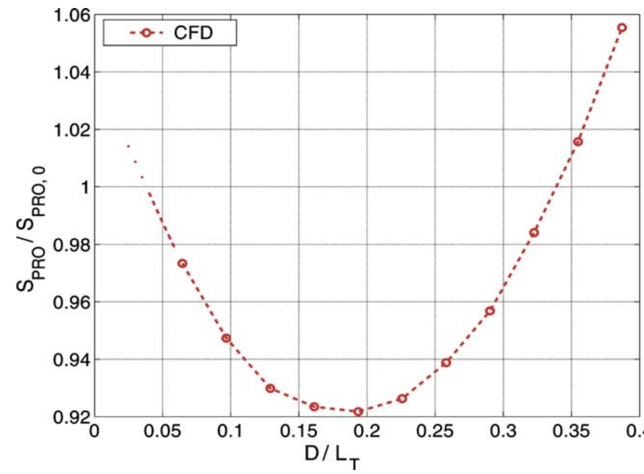

Fig. 4. Total entropy production for various slopes of the twisted tape in heated pipe flow; $S_{PRO,0}$: no tape inserted. CFD [Kock, 2003].

6. Conclusion

It has been shown that entropy production in complex heat transfer problems involving turbulent flows can be calculated in a post-processing procedure. Wall functions for the four different entropy production mechanisms should be incorporated for high Reynolds number flows.

The model presented can be implemented in any CFD code that uses a turbulence model with an ε-equation. It then may serve as a powerful tool to assess the efficiency of certain changes in turbulent flows with heat transfer.

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